Journal of Quality Measurement and Analysis *JQMA 6(2) 2010, 105-117*
Jurnal Pengukuran Kualiti dan Analisis

UNSTEADY MAGNETOHYDRODYNAMIC MIXED CONVECTION STAGNATION POINT FLOW OF A VISCOELASTIC FLUID ON A VERTICAL SURFACE

(Aliran Titik Genangan Olakan Campuran Magnetohidrodinamik Tak Mantap bagi Bendalir Likat-kenyal pada Permukaan Menegak)

KARTINI AHMAD & ROSLINDA NAZAR

ABSTRACT

In this paper, the problem of unsteady magnetohydrodynamic (MHD) viscoelastic fluid flowing towards a stagnation point on a vertical surface is studied. The temperature of the surface is assumed to vary linearly with the distance from the stagnation point. The partial differential equations which governed the flow are transformed into a system of ordinary differential equations, which are then solved numerically by an implicit finite-difference scheme known as the Keller-box method. The effects of the viscoelastic parameter K, mixed convection parameter λ, magnetic parameter M and Prandtl number Pr on the flow and heat transfer characteristics are presented in this paper. The numerical solutions obtained are uniformly valid for all dimensionless time from initial unsteady-state flow to final steady-state flow in the whole spatial region.

Keywords: magnetohydrodynamic (MHD); mixed convection; unsteady stagnation point flow; vertical surface; viscoelastic fluid

ABSTRAK

Dalam makalah ini, masalah aliran bendalir likat-kenyal magnetohidrodinamik (MHD) tak mantap yang mengalir ke arah suatu titik genangan pada permukaan tegak dikaji. Suhu permukaan dianggap berubah secara linear terhadap jarak daripada titik genangan. Persamaan pembezaan separa yang menakluk aliran kemudiannya dijelmakan kepada sistem persamaan pembezaan biasa, yang seterusnya diselesaikan secara berangka menggunakan skema beza terhingga tersirat yang dikenali sebagai kaedah kotak Keller. Kesan parameter likat-kenyal K, parameter olakan campuran λ, parameter magnetik M dan nombor Prandtl Pr terhadap ciri-ciri aliran dan pemindahan haba dipertimbangkan dalam makalah ini. Penyelesaian berangka yang diperoleh adalah sah secara seragam bagi julat masa tak bermatra bermula daripada aliran awal tak mantap kepada aliran akhir mantap.

Kata kunci: magnetohidrodinamik (MHD); olakan campuran; aliran titik genangan tak mantap; permukaan tegak; bendalir likat-kenyal

1. Introduction

Viscoelastic fluid is one type of second grade fluid (non-Newtonian fluid) that has received much attention recently, as this class of fluid exhibits viscous and elastic-like characteristics when undergoing deformation. Honey, plastic films, and artificial fibers are some examples of viscoelastic fluid. Hence, industrially, this kind of fluid has become one of the topics discussed. Pioneering work by Oldroyd (1950), Beard and Walters (1964a) and Rajagopal *et al.* (1984), who have developed the boundary layer theory for the second grade fluid have motivated many researchers to really explore this kind of fluid with various situations. It seems that the boundary layer equations for this fluid are an order higher than those for the Newtonian (viscous) fluid and the adherence boundary conditions are insufficient to determine the solution of these equations completely. Therefore, Rajagopal (1984, 1995) and Rajagopal and Kaloni (1989) have done further investigation on this matter.

The study of a non-Newtonian fluid flow in the region of stagnation point has been done by several authors (Srivatsava 1958; Rajeswari & Rathna 1962; Beard & Walters 1964b; Garg & Rajagopal 1990; Ariel 2002; Mahapatra & Gupta 2004). Ayub *et al*. (2008) studied the stagnation point flow of viscoelastic fluid towards a stretching sheet, Li *et al.* (2009) investigated the case of oblique stagnation point flow of a viscoelastic fluid with the effect of heat transfer and very recently, Labropulu *et al.* (2010) considered the two-dimensional stagnation point flow of a viscoelastic second-grade fluid over a stretching surface with heat transfer and Robert *et al.* (2010) established the existence and uniqueness results over the semi-infinite interval [0,1) for a class of nonlinear fourth order ordinary differential equations arising in the hydromagnetic stagnation point flow of a second grade fluid over a stretching sheet. Further, the effect of mixed convection stagnation point flow in a viscoelastic fluid adjacent to a vertical surface has been studied by Hayat *et al*. (2008), and Anwar *et al*. (2008) considered the flow of a viscoelastic fluid over a horizontal circular cylinder, while the steady MHD mixed convection flow of a viscoelastic fluid in the vicinity of two-dimensional stagnation point with magnetic field has been investigated by Kumari and Nath (2009).

Most of the recent researches on convective flows in non-Newtonian fluids have been directed to the problems of steady cases. However, unsteady convective boundary layer flow problems have not, so far, received as much attention. Therefore, there are few literatures reported on problems that took into account the time factor. Several unsteady problems were done by, for example, Nazar *et al.* (2004), Ishak *et al.* (2006) and Hassanien and Al-Arabi (2009) for the case of mixed convection flow in viscous fluid and porous medium. Lok *et al*. (2006) studied the unsteady mixed convection flow for the case of micropolar fluid, whilst unsteady boundary layer for second-grade fluid flow can be found in the paper by Labropulu *et al.* (2003). Sujit and Pop (2006) investigated the unsteady boundary layer free convection flow of an incompressible electrically conducting viscoelastic second-order fluid over a vertically permeable flat plate, where temperature and concentration differences are responsible for the convective buoyancy current while Qi and Xu (2007) considered the unsteady flow of viscoelastic fluid with the fractional derivative Maxwell model in a channel.

Problems of mixed convection flow near the stagnation point with associated MHD has been pointed out by Abdelkhalek (2006), who presented problem of steady two-dimensional laminar magnetohydrodynamic-mixed convection owing to the stagnation flow against a heated vertical semi-infinite permeable surface for viscous fluid. Ishak *et al*. (2008) considered a steady MHD flow towards a stagnation point on a vertical surface immersed in a micropolar fluid and recently, Ishak *et al*. (2010) investigated the problem of MHD mixed convection boundary layer flow of a viscous and electrically conducting fluid near the stagnation-point on a vertical permeable surface for viscous fluid flow. On the other hand, Hayat *et al.* (2010) has presented the analytical analysis of steady MHD two-dimensional mixed convection boundary layer flow of a viscous and incompressible fluid near the stagnation-point on a vertical stretching surface embedded in a fluid-saturated porous medium and thermal radiation using the homotopy analysis method (HAM).

Motivated by the work of Hayat *et al*. (2008) and Lok *et al*. (2006), we investigate in this paper the unsteady MHD mixed convection boundary layer flow of a viscoelastic fluid near the stagnation point on a vertical surface. The transformed governing partial differential equations in two variables are solved numerically using the Keller-box method for certain values of the viscoelastic parameter K, magnetic parameter M, mixed convection parameter λ and Prandtl number Pr.

2. Basic equations

Consider the mixed convection boundary layer stagnation point flow over a semi-infinite vertical surface, which is placed in a viscoelastic fluid of uniform ambient temperature T_∞. It is assumed that a uniform magnetic field of strength B_o is applied in the positive y-direction and the surface of the plate is heated or cooled to a variable temperature $T_w(x)$, where $T_w(x) > T_\infty$ is for a heated plate and $T_w(x) < T_\infty$ is for a cooled plate. It is also assumed that at time t=0 the outside flow starts in motion impulsively from rest towards the plate with a steady velocity $u_e(x)$, as shown in Fig. 1. Figures 1(a) and 1(b) illustrate the assisting and opposing flows, respectively.

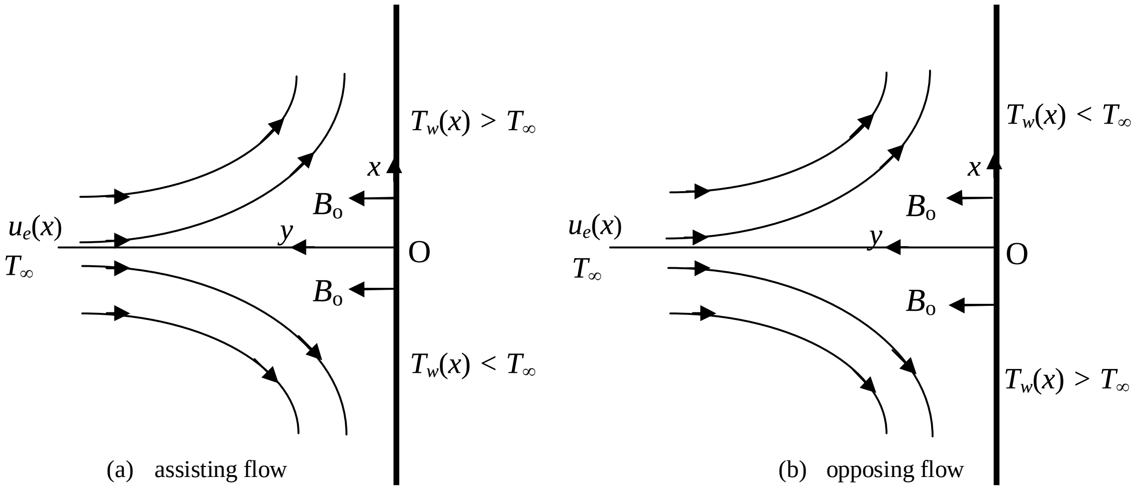

Figure 1: Physical model and coordinate system

The flow in the neighbourhood of the stagnation line has the same characteristics irrespective of the shape of the body (Hiemenz 1911). It is further assumed that the temperature of the plate $T_w(x)$ varies linearly with the distance x along the plate. Thus the plate temperature and the condition far from the plate is assumed to be given by

$$T_w(x) = T_\infty + T_0\left(\frac{x}{L}\right), \qquad u_e(x) = U_e\left(\frac{x}{L}\right), \tag{1}$$

where U_e is the reference velocity, L is the characteristic length and $T_0 > 0$ is a reference temperature. In the absence of heat generation and viscous dissipation, along with the Boussinesq approximation, the boundary layer equations are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + k_0\left(u\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right) \\ \pm g\beta\left(T - T_\infty\right) + \sigma\frac{B_0^{\,2}}{\rho}\left(u_e - u\right), \end{aligned} \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

subject to the boundary conditions

$$\begin{aligned}
t<0: &\quad u(x,y)=v(x,y)=0,\quad T(x,y)=T_\infty \qquad \text{any}\quad x,y\\
t\ge 0: &\quad u(x,0)=v(x,0)=0,\quad T(x,0)=T_w(x)=T_\infty+T_0\left(\frac{x}{L}\right),\\
&\quad u(x,\infty)=u_e(x)=U_e\left(\frac{x}{L}\right),\quad \frac{\partial u(x,\infty)}{\partial y}=0,\quad T(x,\infty)=T_\infty,\qquad x\ge 0,
\end{aligned} \tag{5}$$

where u and v are the velocity components in the x-and y-directions, respectively, t denotes the time, T, ν, g, β, α, σ, ρ, B_0 and k_0 are the temperature, kinematic viscosity, gravity acceleration, thermal expansion coefficient, thermal diffusivity, electrical conductivity, density, magnetic field and viscoelastic parameter, respectively. The $\pm$ sign in Eq. (3) corresponds to the assisting and opposing flows, respectively. Introducing the new variables as follows (Williams & Rhyne 1980), where prime denotes differentiation with respect to η:

$$\begin{gathered}
\eta=\left(\frac{U_e}{L\nu}\right)^{1/2} y\xi^{-1/2},\quad \xi=1-\exp(-t^*),\quad t^*=\left(\frac{U_e}{L}\right)t,\quad u(x,y,t)=U_e\left(\frac{x}{L}\right)f'(\xi,\eta),\\
v(x,y,t)=-\left(\frac{U_e\nu}{L}\right)^{1/2}\xi^{1/2}f(\xi,\eta),\quad T(x,y,t)=T_\infty+T_0\left(\frac{x}{L}\right)\theta(\xi,\eta).
\end{gathered} \tag{6}$$

Substituting (6) into Eqs. (3) and (4) yields

$$\begin{aligned}
&\frac{1}{2}\eta(1-\xi)f''+\left(1-f'^2+f\,f''\right)\xi+K\left(2f'f'''-f''^2-f\,f^{iv}\right)\\
&\qquad\qquad+f'''+\lambda\xi\theta+M\xi(1-f')=\xi(1-\xi)\frac{\partial f'}{\partial\xi},
\end{aligned} \tag{7}$$

$$\frac{1}{\Pr}\theta''+\frac{1}{2}\eta(1-\xi)\theta'+\xi\left(f\theta'-f'\theta\right)=\xi(1-\xi)\frac{\partial\theta}{\partial\xi}, \tag{8}$$

for $0<\xi\le 1$. Here $\lambda=\dfrac{Gr}{\mathrm{Re}^2}$ is the mixed convection parameter, where $Gr=\dfrac{g\beta T_0L^3}{\nu^2}$ and $\mathrm{Re}=\dfrac{U_eL}{\nu}$ are the Grashof and Reynolds numbers, respectively. It is worth mentioning that $\lambda>0$, $\lambda<0$ and $\lambda=0$ correspond to the assisting flow, opposing flow and forced convection flow, respectively. On the other hand, M and $K(\ge 0)$ are the dimensionless magnetic and viscoelastic parameters, respectively, given by

$$M=\frac{\sigma B_0^2}{\rho}\frac{L}{U_e},\qquad K=k_0\frac{U_e}{\nu L}. \tag{9}$$

The boundary conditions (5) become

$$\begin{aligned}
&f(\xi,0)=f'(\xi,0)=0,\quad \theta(\xi,0)=1,\\
&f'\to 1,\ f''\to 0,\ \theta\to 0\qquad \text{as}\quad \eta\to\infty,
\end{aligned} \tag{10}$$

for $0<\xi\le 1$.

The physical quantities of interest in this problem are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho u_e^2}, \quad Nu_x = \frac{x q_w}{\alpha\left(T_w - T_\infty\right)}, \tag{11}$$

where τ_w and q_w are the wall shear stress and the surface heat flux, respectively, which are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} + k_0\left(u\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{12}$$

Substituting variables (6) into (12), we obtain

$$C_f \operatorname{Re}_x^{1/2} = \xi^{-1/2} f''(0), \quad Nu_x \operatorname{Re}_x^{-1/2} = -\xi^{-1/2}\theta'(0). \tag{13}$$

for $0 < \xi \le 1$. Equations (7) and (8) subject to boundary conditions (10) are coupled nonlinear parabolic partial differential equations. Hence, we can obtain some particular cases of this problem.

2.1. *Initial unsteady flow*

For early unsteady flow, we have $\xi \to 0\,(t^* = 0)$, thus Eqs. (7) and (8) reduce to

$$\frac{1}{2}\eta\, f'' + K\left(2 f' f''' - f''^2 - f\, f^{iv}\right) + f''' = 0, \tag{14}$$

$$\frac{1}{\Pr}\theta'' + \frac{1}{2}\eta\,\theta' = 0 \tag{15}$$

and the boundary conditions (10) become

$$\begin{aligned} &f(0) = f'(0) = 0, \quad \theta(0) = 1, \\ &f' \to 1, \ f'' \to 0, \ \theta \to 0 \qquad \text{as} \quad \eta \to \infty \end{aligned} \tag{16}$$

2.2. *Final steady-state flow*

For this case, $\xi = 1\,(t^* \to \infty)$, Eqs. (7) and (8) take the following form:

$$1 - f'^2 + f\, f'' + K\left(2 f' f''' - f''^2 - f\, f^{iv}\right) + f''' + \lambda\theta + M\left(1 - f'\right) = 0, \tag{17}$$

$$\theta'' + \Pr\left(f\,\theta' - f'\theta\right) = 0, \tag{18}$$

subject to the same boundary conditions (16).

3. Results and Discussion

The nonlinear partial differential equations (7) and (8) subject to boundary conditions (10) have been solved numerically using an implicit finite-difference scheme known as the Keller-box method as described in the book by Cebeci and Bradshaw (1988).

Tables 1 and 2 present the numerical values of the skin friction coefficient $C_f \mathrm{Re}_x^{1/2} = f''(0)$ and the local Nusselt number $Nu_x \mathrm{Re}_x^{-1/2} = -\theta'(0)$, respectively, when ξ=1 (final steady-state flow) for various values of the viscoelastic parameter K when Pr=0.2 and Pr=10 for the case of assisting flow ($\lambda > 0$) and opposing flow ($\lambda < 0$). In order to validate the accuracy of the present method, the results for the final steady-state flow obtained were compared with those of Hayat *et al*. (2008) and it is found that they are in good agreement. Therefore, we are confident that the developed code used in this study is suitable to solve the present problem discussed in this paper.

Table 1: Values of $C_f\, Re_x^{1/2}$ for various values of K and Pr when λ=0.2, -0.2. Results shown in ( ) are those of Hayat *et al*. (2008)

M	K	Pr=0.2		Pr=10	
		λ=0.2	λ= -0.2	λ=0.2	λ= -0.2
0	0.2	1.1559 (1.1559)	0.9561 (0.9558)	1.1058	1.0096
	1	0.8174 (0.8174)	0.6844 (0.6844)	0.7905	0.7141
	2	0.6472 (0.6474)	0.5432 (0.5432)	0.6291	0.5636
1	0.2	1.4554	1.2948	1.4171	1.3346
	1	1.0513	0.9470	1.0312	0.9682
	2	0.8419	0.7617	0.8287	0.7758
10	0.2	3.0220	2.9401	3.0067	2.9555
	1	2.2416	2.1901	2.2338	2.1979
	2	1.8193	1.7805	1.8143	1.7856

Table 2: Values of $Nu_x\, Re_x^{-1/2}$ for various values of K and Pr number when λ=0.2, -0.2. Results shown in ( ) are those of Hayat *et al.* (2008)

M	K	Pr=0.2		Pr=10	
		λ=0.2	λ= -0.2	λ=0.2	λ= -0.2
0	0.2	-0.4261 (-0.4261)	-0.4096 (-0.4094)	-1.7909	-1.7564
	1	-0.3919 (-0.3920)	-0.3784 (-0.3785)	-1.6090	-1.5764
	2	-0.3696 (-0.3698)	-0.3575 (-0.3578)	-1.4957	-1.4641
1	0.2	-0.4403	-0.4288	-1.9159	-1.8907
	1	-0.4085	-0.3993	-1.7320	-1.7096
	2	-0.3872	-0.3791	-1.6160	-1.5950
10	0.2	-0.4832	-0.4795	-2.3336	-2.3243
	1	-0.4585	-0.4555	-2.1334	-2.1259
	2	-0.4408	-0.4382	-2.0042	-1.9975

From Table 1, it can be seen that the values of $C_f \mathrm{Re}_x^{1/2}$ decrease as the viscoelastic parameter K increases regardless whether the flow is assisting or opposing. The same phenomenon happened for $Nu_x \mathrm{Re}_x^{-1/2}$ as can be seen from Table 2. Increasing Pr and heating the plate causing $C_f \mathrm{Re}_x^{1/2}$ to decrease and the opposite trends are observed for absolute values of

$Nu_x \mathrm{Re}_x^{-1/2}$. On the other hand, cooling the plate will increase the values of $f''(0)$ and $-\theta'(0)$. The magnetic parameter *M* gives huge impact to the flow as it increases the values of $f''(0)$ and $-\theta'(0)$ for both assisting and opposing flows.

Figures 2 and 4 show the velocity profiles of the final steady-state flow (ξ=1) for various values of the viscoelastic parameter *K* and magnetic parameter *M* when *Pr*=10 for assisting and opposing flows, respectively. For a particular value of *K*, the velocity boundary layer thickness increases monotonically with η, and becomes unity at the outside of the boundary layer, which actually satisfies the boundary condition $f'(\infty)\rightarrow 1$. It is also illustrated that for a specific value of *K* as the magnetic parameter *M* increases, the velocity thickness decreases.
The temperature profiles of the final steady-state flow are shown in Figs. 3 and 5 for both assisting and opposing flows, respectively. It is observed that as *K* increases, the temperature profile increases. The thermal boundary layer thickness is slightly smaller when *M*=10 compared to *M*=1. This justify that the magnetic parameter *M* reduces the thickness of the thermal boundary layer. Comparatively, the velocity profiles for the assisting and opposing flows seem alike and thus, it can be concluded that the effects of the plate either being cooled or heated are not significant. The same trend also occurs for the temperature profiles.

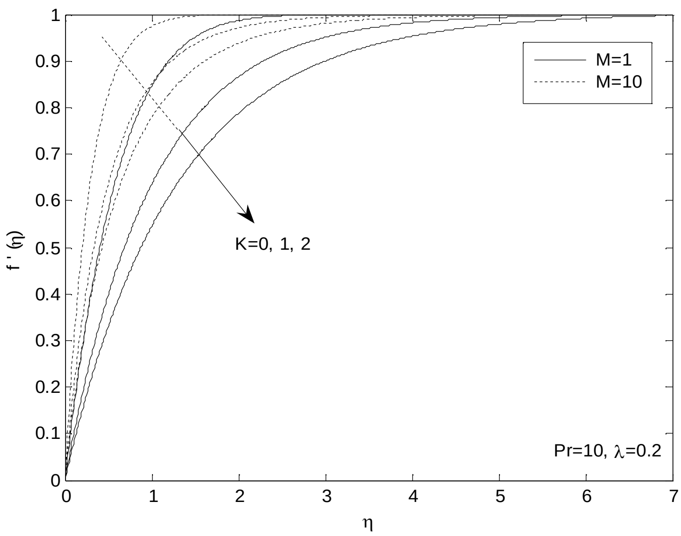

Figure 2: Velocity profiles of the final steady-state flow for various *K* and *M* when *Pr*=10 for λ=0.2 (assisting flow)

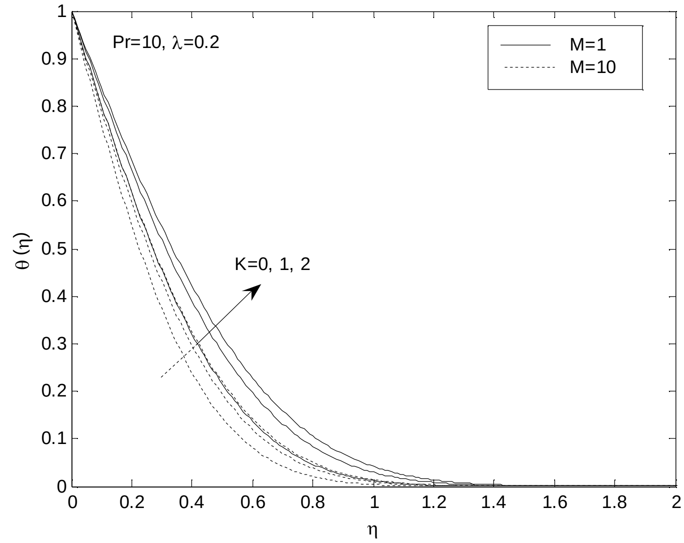

Figure 3: Temperature profiles of the final steady-state flow for various *K* and *M* when *Pr*=10 for λ=0.2 (assisting flow)

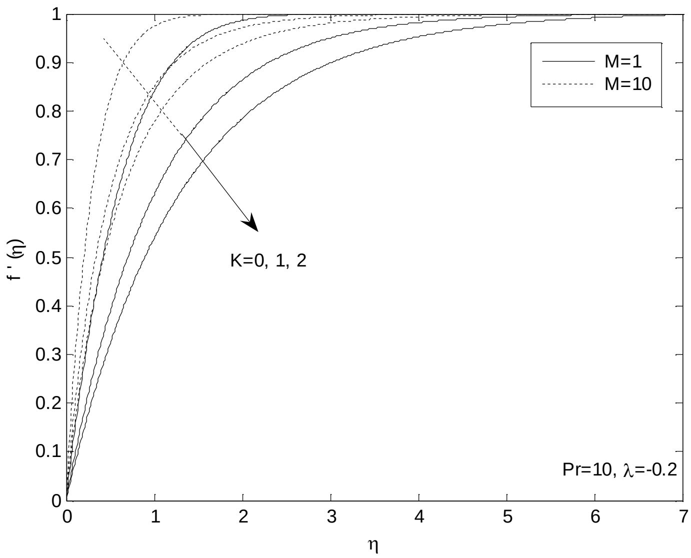

Figure 4: Velocity profiles of the final steady-state flow for various *K* and *M* when *Pr*=10 for λ=-0.2 (opposing flow)

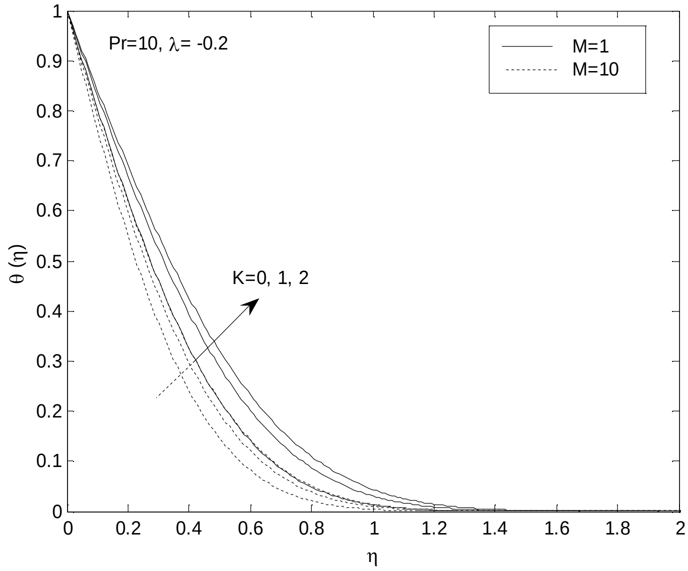

Figure 5: Temperature profiles of the final steady-state flow for various K and M when Pr=10 for λ=-0.2 (opposing flow)

Figures 6 and 7 show the velocity and temperature profiles when Pr=0.7 and K=1 by considering various values of the magnetic parameter M. It is observed that the thickness of the velocity and thermal boundary layer decreases as M increases, and valid for both assisting and opposing flows. Again, the effects of the plate either being cooled or heated are not significant, especially for large values of M.

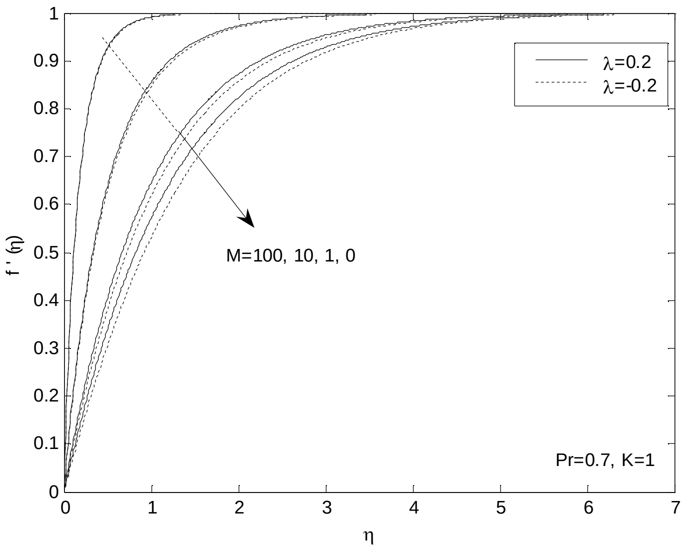

Figure 6: Velocity profiles of the final steady-state flow for various M when Pr=0.7 and K=1 for λ=0.2 (assisting flow) and λ=-0.2 (opposing flow).

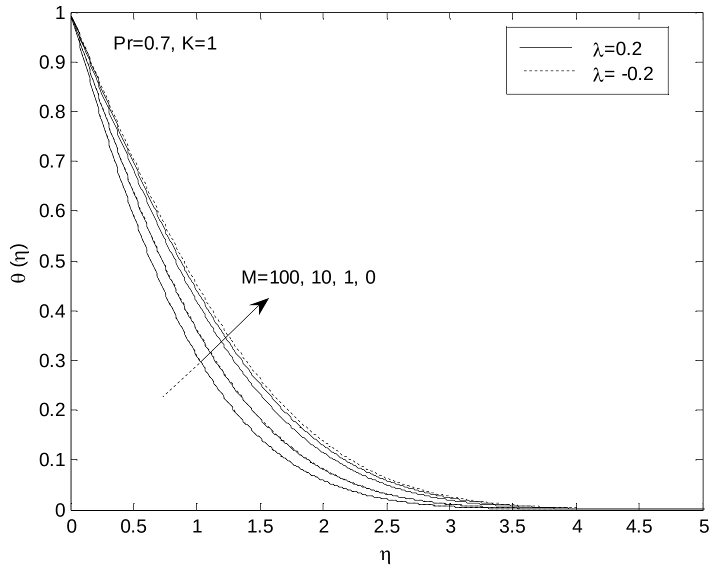

Figure 7: Temperature profiles of the final steady-state flow for various *M* when *Pr*=0.7 and *K*=1 for λ=0.2 (assisting flow) and λ=-0.2 (opposing flow).

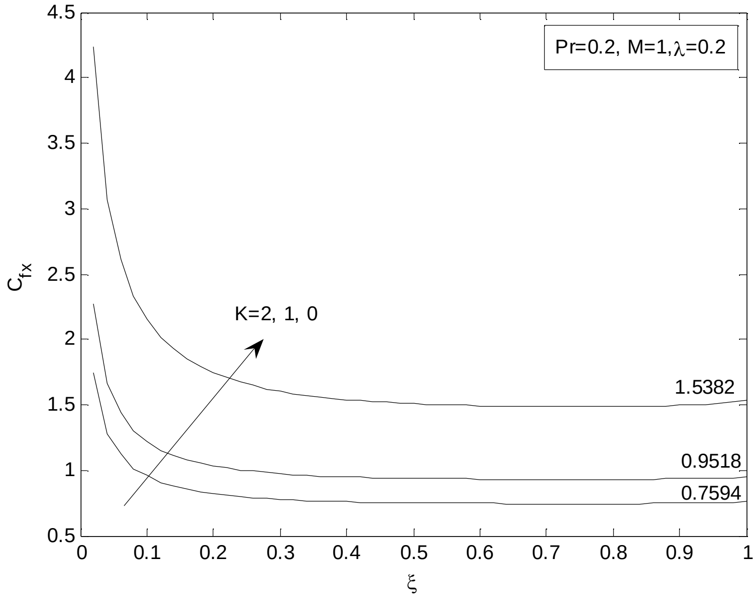

Figure 8: Variation of the skin friction coefficient with ξ for various *K*.

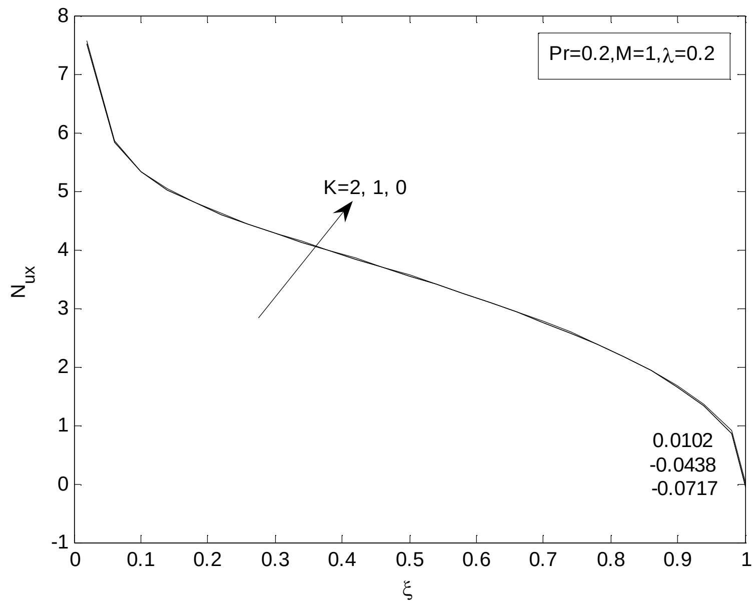

Figure 9: Variation of the local Nusselt number with ξ for various *K*.

Figures 8 and 9 present the variation of the skin friction coefficient $C_f \mathrm{Re}_x^{1/2}$ and the local Nusselt number $Nu_x \mathrm{Re}_x^{-1/2}$, respectively, as a function of ξ for various values of *K*. At the start of the motion $t = 0$ or $\xi \to 0$ (initial unsteady-state flow), both the skin friction coefficient and the local Nusselt number have large magnitude (due to impulsive motion) and decrease monotonically and finally reach the steady-state value $\xi = 1\,(t \to \infty)$ .The flow transition is very smooth from small-time solution to the large-time solution. The values of $C_f \mathrm{Re}_x^{1/2}$ and $Nu_x \mathrm{Re}_x^{-1/2}$ seem to decrease when the viscoelastic parameter *K* increases, while the viscoelastic parameter *K* is not really affecting the value of $Nu_x \mathrm{Re}_x^{-1/2}$ at $\xi = 1$ since the changes in $Nu_x \mathrm{Re}_x^{-1/2}$ are comparatively small for variation in *K*.

4. Conclusions

In the present study, we have investigated theoretically the unsteady MHD mixed convection flow of a viscoelastic fluid near the stagnation point on a vertical surface. Both the assisting flow and opposing flow situations are considered. Numerical computation has been carried out to study the effects of the material parameter *K*, magnetic parameter *M*, mixed convection parameter λ and Prandtl number *Pr* on the skin friction coefficient, the local Nusselt number, as well as the velocity and temperature profiles. Results are presented in tables and figures for certain parameter conditions. From the solutions of the skin friction and heat transfer coefficients it is found that there is a smooth transition from the small-time solution (initial unsteady-state) to the large-time solution (final steady-state).

Acknowledgment

The financial support received in the form of a fundamental research grant scheme (FRGS) from the Ministry of Higher Education Malaysia is greatly acknowledged. The authors would also like to thank the reviewer for the valuable comments and suggestions.

References

Abdelkhalek M.M. 2006. The skin friction in the MHD mixed convection stagnation point with mass transfer. *Int. Comm. Heat Mass Transfer* **33**(2): 249-258.

Anwar I., Amin N. & Pop I. 2008. Mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder. *Int. J. Non-Linear Mech.* **43**(9): 814-821.

Ariel P. D. 2002. On extra boundary condition in the stagnation point flow of a second-grade fluid. *Int. J. Eng. Sci.* **40**: 145–162.

Ayub M., Zaman H., Sajid M. & Hayat T. 2008. Analytical solution of stagnation-point flow of a viscoelastic fluid towards a stretching surface. *Commun. Nonlinear Sci. Numer. Simulation* **13**(9): 1822-1835.

Beard D.W. & Walters K. 1964a. Elasto-viscous boundary layer flows. *Proc. Camb. Phil. Soc.* **60**: 667–674.

Beard D.W. & Walters K. 1964b. Elastco-viscous boundary layer flows. Part I: two dimension flow near a stagnation point. *Proc. Camb. Phil. Soc.* **60**: 667-674.

Cebeci T. & Bradshaw P. 1988. *Physical and Computational Aspects of Convective Heat Transfer*. New York: Springer.

Garg V.K. & Rajagopal K.R. 1990. Stagnation point flow of a non-Newtonian fluid. *Mech. Res. Comm.* **17**: 415-421.

Hassanien I.A. & Al-Arabi T.H. 2009. Non-Darcy unsteady mixed convection flow near the stagnation point on a heated vertical surface embedded in a porous medium with thermal radiation and variable viscosity. *Commun. Nonlinear Sci. Numer. Simulation* **14**(4): 1366-1376.

Hayat T., Abbas Z. & Pop I. 2008. Mixed convection in the stagnation point flow adjacent to a vertical surface in a viscoelastic fluid. *Int. J. Heat Mass Transfer* **51**(11-12): 3200-3206.

Hayat T., Abbas Z., Pop I. & Asghar S. 2010. Effects of radiation and magnetic field on the mixed convection stagnation-point flow over a vertical stretching sheet in a porous medium. *Int. J. Heat Mass Transfer* **53**(1-3): 466-474.

Hiemenz K. 1911. Der Grenzschit an einem in den gleichförmigen Flüssigkeitsstrom eingetauchten geraden Kreiszylinder. *Dingl. Poly-tech.* **32**: 321-410.

Ishak A., Nazar R. & Pop I. 2006. Unsteady mixed convection boundary layer flow due to a stretching vertical surface. *Arabian J. Sci. & Engng.* **31**: 165–182.

Ishak A., Nazar R. & Pop I. 2008. Magnetohydrodynamic (MHD) flow of a micropolar fluid towards a stagnation point on a vertical surface. *Comp. & Math. with Appl.* **56**(12): 3188-3194.

Ishak A., Nazar R., Bachok N. & Pop I. 2010. MHD mixed convection flow near the stagnation point on a vertical permeable surface. *Physica A: Statistical Mech. & Appl.* **389**(1): 40-46.

Kumari M. & Nath G. 2009. Steady mixed convection stagnation-point flow of upper convected Maxwell fluids with magnetic field. *Int. J. Non-Linear Mech.* **44**(10): 1048-1055.

Labropulu F., Xu X. & Chinichian M. 2003. Unsteady stagnation point flow of a non-newtonian second-grade fluid. *Int. J. Maths. & Mathematical Sci.* **2003**(60): 3797-3807.

Labropulu F., Li D. & Pop I. 2010. Non-orthogonal stagnation-point flow towards a stretching surface in a non-Newtonian fluid with heat transfer. *Int. J. Thermal Sciences* **49**(6): 1042-1050.

Li D., Labropulu F. & Pop I. 2009. Oblique stagnation-point flow of a viscoelastic fluid with heat transfer. *Int. J. Non-linear Mech.* **44**: 1024-1030.

Lok Y.Y., Amin N. & Pop I. 2006. Unsteady mixed convection flow of a micropolar fluid near the stagnation point on a vertical surface. *Int. J. Thermal Sciences* **45**(12): 1149-1157.

Mahapatra T.R. & Gupta A.S. 2004. Stagnation-point flow of a viscoelastic fluid towards a stretching surface. *Int. J. Non-Linear Mech.* **39**: 811-820.

Nazar R., Amin N. & Pop I. 2004. Unsteady mixed convection boundary layer flow near the stagnation point on a vertical surface in a porous medium. *Int. J. Heat Mass Transfer* **47**(12-13): 2681-2688.

Oldroyd J.G. 1950. On the formulation of rheological equations of state. *Proc. Roy. Soc. London A* **200**: 523–541.

Qi H. & Xu M. 2007. Unsteady flow of viscoelastic fluid with fractional Maxwell model in a channel. *Mech. Res. Comm.* **34**(2): 210-212.

Rajagopal K.R. 1984. On the creeping flow of the second-order fluid. *J. Non-Newtonian Fluid Mech.* **15**: 239–246.

Rajagopal K.R. 1995. On boundary conditions for fluids of the differential type. In: Sequiera A. (Ed.). *Navier–Stokes Equations and Related Non-Linear Problems.* New York: Plenum.

Rajagopal K.R., Na T.Y. & Gupta A.S. 1984. Flow of a viscoelastic fluid over a stretching sheet. *Rheol. Acta* **23**: 213–215.

Rajagopal K.R. & Kaloni P.N. 1989. Some remarks on boundary conditions for flows of fluids of the differential type. In: Graham G.A.C. & Malik S.K. (Eds.). *Continuum Mechanics and its Applications*. New York: Hemisphere.

Rajeswari G. & Rathna S.L. 1962. Flow of a particular class of non-Newtonian viscoelastic and visco-inelastic fluids near a stagnation point. *Z. Angew. Math. Phys. (ZAMP)* **13**: 43–57.

Robert A., Gorder V. & Vajravelu K. 2010. Hydromagnetic stagnation point flow of a second grade fluid over a stretching sheet. *Mech. Res. Comm.* **37**(1): 113-118.

Srivatsava A.C. 1958. The flow of a non-Newtonian liquid near a stagnation point. *Z. Angew. Math. Phys.* **9**: 80–84.

Sujit K.K. & Pop I. 2006. Unsteady free convection viscoelastic boundary layer flow past a vertical porous plate with internal heat generation/absorption. *Int. J. Fluid Mech. Res.* **33**(6): 500-522.

Williams J.C. & Rhyne T.H. 1980. Boundary layer development on a wedge impulsively set into motion. *SIAM J. Appl. Math.* **3**: 215-224.

Centre for Foundation Studies
International Islamic University Malaysia
46350 Petaling Jaya
Selangor DE, MALAYSIA
E-mail: katzdany@yahoo.com

Pusat Pengajian Sains Matematik
Fakulti Sains dan Teknologi
Universiti Kebangsaan Malaysia
43600 UKM Bangi
Selangor DE, MALAYSIA
E-mail: rmn@ukm.my[*]

[*] Corresponding author